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Rev. T. E. R. PHILLIPS, PRESIDENT, in the Chair.

Raffaele Invrea, Royal Observatory, Pino Torinese, Italy;
Charles Mowat, Hall Cottage, St. Mary's Holm, by Kirkwall, Orkney;
Francis John Sellers, M.I.M.E., 42 Church Crescent, London, N. 10;
Henry Frederic Tiarks, Foxbury, Chislehurst, Kent;
George Mortimer Townsend, Hawera, New Zealand; and
Alfred Vowles, Bampton Street, Minehead, Somerset,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Henry George Scott Barrett, Radcliffe Observatory, Oxford (proposed by H. Knox-Shaw);

D. G. A. Dyson, B.A., Kenwood, Loxley Lane, Stratford-on-Avon (proposed by A. S. Eddington);

Harold Jameson, B.Sc., Assistant Astronomer, Colombo Observatory, Ceylon (proposed by A. J. Bamford);

Bertram F. Jearey, c/o Divine Gates & Co., St. George's Street, Cape Town, S. Africa (proposed by L. J. Comrie);

Mohamed Abdur Rahman Khan, A.R.C.S., B.Sc., F.P.S.L., Principa of Osmania University College, Hyderabad, Deccan, India (proposed by A. Fowler);

Sydney Royston Pike, M.A., Physics Department, The University, Leeds (proposed by R. Stoneley);
Sidney Herbert Pope, 31 Westbury Avenue, Wembley (proposed by W. A. Parr);
Rev. John Pierce Price, North View, Dolcliffe Road, Mexborough, Yorks (proposed by M. Proctor);
R. O. Redman, B.A., Isaac Newton Student, St. John's College, Cambridge (proposed by A. S. Eddington);
John J. Richardson, Kurrawyba, Upper Spit Road, Mosman, N.S.W., Australia (proposed by Hugh Wright);
William Henry Robinson, Radcliffe Observatory, Oxford (proposed by H. Knox-Shaw);
Abinas Chandra SenGupta, A.C.S. Calcutta Port, 15 Strand Road, Calcutta, India (proposed by H. H. Furner); and
Mario Rodriques de Sousa, Chief Assistant, Observatory, Rio de Janeiro (proposed by A. S. Eddington).

Seventy-three presents were announced as having been received since the last meeting, including, amongst others :—

Tycho Brahe, Opera Omnia, Tomus IX. (presented by Admiral Dreyer).
Two stereoscopic transparencies of Jupiter, photographed (*a*) by ultra-violet light, (*b*) by extreme red light (presented by Dr. W. H. Wright, Lick Observatory).

Mr. P. J. Melotte and Mr. C. O. Bartrum were appointed Auditors of the Treasurer's Accounts for 1927.

The Solar Rotation and the Einstein Displacement derived from Measures of the H and K Lines in Prominences. By J. Evershed, F.R.S.

In *Monthly Notices*, **85**, 608, values of the solar rotation are given derived from spectrographic measures in the reversing layer, the chromosphere, and the prominences. These show a general increase of angular speed with height above the photosphere; the prominences with a mean height of 46″ yielding a daily angular motion of 16°·8 at the equator, corresponding to a complete rotation in about twenty-one days.

The prominence results were derived from measures of the H and K lines in 61 prominences photographed in the years 1908–11 at Kodaikanal. The result was considered to be somewhat doubtful, owing

to the large individual motions of the prominences, even when those of a quiet type were selected for the measures.

During the past year about 200 spectra of prominences have been photographed at Ewhurst, and the results of measures of a selected series of these plates give a striking confirmation of the previous results, and they are of interest also in giving a value of the general displacement towards red (Einstein effect), free from the disturbing effects which may alter the wave-lengths of lines in the reversing layer. On the other hand, the results are subject to a larger uncertainty than is the case with measures of lines in the reversing layer, owing to the very considerable random motions of the prominences, movements which disturb all regions of the Sun's atmosphere, and, as recorded in my paper referred to above, appear to increase in amplitude with height above the photosphere. It is on the assumption that these motions may be treated as accidental, and are not systematic with reference to the direction of the Earth, that the results of this paper are based.

Before giving a *résumé* of the measures, I will briefly describe the instrumental equipment and the methods employed in photographing the spectra and in measuring the plates.

The spectroheliograph of the Pitch Hill Observatory is designed primarily for photographs of the Sun's disc and limb in monochromatic Hα light. It is constructed of parts which I made for use in Kashmir in 1916, and the two prisms of 6-inch aperture and 45° angle, which served in Kashmir for photographs in calcium light with a relatively short camera and single transmission, are now again employed, after having been re-annealed by the Parsons Optical Company and re-figured by Messrs. Hilger. These prisms are now probably the finest that have ever been produced of this size, and give excellent definition if kept under perfectly uniform temperature conditions. The spectroheliograph is auto-collimating; a plane mirror placed behind the two prisms returns the light through them, doubling the dispersion. The collimator is a simple lens of meniscus form and 16 feet focal length. The concave rear surface has the convenient property of returning the reflected white light to a focus at the first slit, and in fact returning it to the Sun through the slit if the squaring-on adjustment is made accurately, but in any case eliminating it from the spectrum. A plane mirror placed near the first slit, and to one side of the incoming beam of light, reflects a short section of spectrum to the second slit of the spectroheliograph, and a second movable mirror is used to intercept a portion of the spectrum and reflect it to a photographic plate. Small secondary reflectors are also used, by means of which the Hα line and the prominences can be observed with an eyepiece, and when used as a spectrograph a part of the spectrum on the redward side of the region to be photographed can also be seen and adjusted to the correct position.

The instrument is mounted in a room the floor of which is about 10 feet below ground-level. It is fed by a 15-inch Cooke cœlostat mounted 4 feet above ground, or 10 feet above the spectroheliograph. A second mirror placed a few feet to the south of the cœlostat reflects the sunlight to a 6-inch achromatic lens, also by Cooke, of 20 feet

9 inches focus. After a third reflection, the image of the Sun formed by this lens is projected on the slit-plate of the spectroheliograph ; it is about 60 mm. in diameter, with the north point always at the top, and the vertical slit always touches the east or west points of the Sun when tangent to the limb.

The instrument is provided with three primary slits of fixed widths, which can be used alternatively : first, a circular slit 0·6 mm. wide, for observing the prominences over a large arc of the limb ; secondly, a slit of the correct curvature and 0·1 mm. wide for spectroheliograph work ; and thirdly, a slit of lesser curvature and 0·03 mm. width for photographing spectra. The curvature is arranged to give perfectly straight spectrum lines, which is a convenience in measuring displacements. Used as a spectrograph, the instrument is very well adapted for solar and terrestrial comparison spectra, owing to the stability of the parts, and especially to the fact that the prisms, mirror, and lens are mounted together in a partly enclosed recess in the end-wall of the room, forming what is virtually a constant-temperature chamber situated about 5 feet underground. Under these conditions, long exposures can be made without fear of shifts due to temperature changes.

The routine work on a clear morning is first to survey the east and west limbs in Hα light with the circular slit, if possible within an hour of sunrise. The positions of prominences observed determine the subsequent proceedings. Photographs in Hα light are then taken, showing the prominences at the limb and on the disc. As soon as these have been developed, the Sun will have risen to a more favourable altitude for work in the H and K region. The narrow slit is substituted for the spectroheliograph slit, and observations are made of the character of the Hα line in any prominence that may be in a favourable position. The height of the prominence above the limb is estimated by means of lines ruled on the slit-plate, spaced at intervals from the slit corresponding with 30″ of arc. The spectroheliograph is then converted into a spectrograph, which involves four simple operations, occupying in all about a minute of time. The image of the Sun on the slit is then observed in a small telescope by reflection from an unsilvered mirror ; it is adjusted so that the slit will bisect the prominence at a known height, the exposure is then begun. After from two to four minutes, the light from the Sun is intercepted, and an image of the iron arc is projected on the slit. A tongue of metal is made to cover the central part of the slit where the prominence was. This is necessary, as the calcium lines are usually present in the iron arc spectrum and would be superposed upon the prominence lines. After a two-minute exposure on the arc, using a ground-glass diffusing screen in front of the slit, exposure on the prominence is resumed, and continued for the same number of minutes as before. Guiding is necessary in all exposures exceeding a minute in duration, and this is easily effected by means of a large screw, which operates the moving parts of the spectroheliograph against a 50-lb. weight : this simultaneously moves the 6-inch object-glass, which is mounted on a carriage running on rails. In this way a very delicate and satisfactory guiding is effected.

For prominences found at some position-angle other than about 90° or 270° the slit has to intersect a portion of the Sun's disc, and a more or less inclined or radial section of the prominence is obtained. In this case the height at any point in the spectral image can be ascertained from its distance from the edge of the solar spectrum, and the angle at which the slit and limb intersect.

The heliographic latitudes are obtained from measures of the spectroheliograph images photographed earlier in the day.

The spectra have a dispersion at H of 1·3 mm. to the angstrom, and in these high-dispersion prism spectra the dispersion may be assumed to vary linearly with the wave-length over a length of about 50 angstroms: the reduction of the measures is therefore as simple as with grating spectra.

Numerous iron arc lines are shown on the plates, but it is only necessary to measure four of these to determine the scale and the factors necessary in computing the apparent wave-lengths of H and K. The following lines, which are not subject to appreciable pole-effect, have been used throughout:—

3920·261,
3930·300,
3969·262,
4005·248.

In some plates the line 3969 is over-exposed, and when this occurs the line 3967·424 is substituted. These lines are from the list of Tertiary Standards published in the *Transactions of the Astronomical Union*, **1**, 43.

The normal wave-lengths in air of H and K are assumed to be as determined by St. John,* viz.:

K 3933·667,
B 3968·476.

These depend on interferometer measures by Fabry and Buisson of iron lines in the neighbourhood, but no correction seems necessary to reduce them to the tertiary standards quoted above.

The measures have been made on a remodelled Hilger micrometer, in which the screw moves a carriage to which the plate is clamped and the microscope of special design is fixed.

In measuring the plates, I have found it necessary to classify the spectra according to the general character of the H and K emission lines. In Class A the lines are narrow, well-defined, and straight; in Class B they are either broad or diffuse, or irregularly bent and displaced, so that measures cannot be made with satisfactory precision. In Class C the displacements and irregularities are so great that only rough estimates can be made of the limiting velocities of the flying masses of gas. These plates are useless for the purpose of this research, and those of Class B have very little weight unless an enormous number is averaged.

The displacements of the prominence lines compared with the above values of H and K are tabulated separately for eastern and

* *Astrophysical Journal*, **31**, 152.

western prominences. It is then found that the western shifts almost invariably exceed the eastern by a considerable amount, half the difference W−E giving the general shift of these lines towards red, whilst half the sum of the shifts W+E gives a value of the solar rotation.

Rotation Results.—Of the whole number of spectra photographed, 92 are of Class A, 45 representing the east limb and 47 the west. The latitudes range from the equator to latitude 44°. In Table I. the mean shifts are given in angstroms of all the western and all the eastern prominences of Class A. The resulting rotation velocity in km./sec. is also given, with the correction to reduce it to sidereal velocity.

TABLE I.

Mean Shifts from all Measures of Class A Prominences.

	K.	H.	
West . . .	+·0420 A	+·0424 A	
East . . .	−·0242 A	−·0251 A	
Half sum . .	·0331	·0337	
Mean	·0334 A		
Equivalent to			2·67 km./sec.
Corrections for revolution of Earth and inclination of Sun's axis			+ ·14 ,,
Sidereal velocity			V=2·81 km./sec.

Mean latitude 18°.
Mean height 29″.

The sidereal velocity here shown is about 1 km./sec. in excess of the normal velocity in the reversing layer at latitude 18°, and, allowing for the mean height of 29″, it corresponds with a daily angular speed of 20°.

A subdivision of the results into four zones of latitude brings out irregularities, due to the random motions of the prominences : it also reveals a tendency to a polar retardation of angular speed, although a considerable modification in the figures would probably result from a larger number of measures. Omitting the separate values of H and K in angstroms, and tabulating only the corrected values, V, of the observed velocities, and the daily angular motion, ξ, results are obtained as in Table II.

TABLE II.

Rotation in different Latitudes at a Mean Height of about 28″ in each case.

No. of Spectra	ϕ.	V.	ξ Prominences.	Spots.
	°		° °	°
33	8	3·17 km./sec.	22·1±1·1	14·4
25	18	2·29 ,,	16·6±0·8	14·1
26	25	2·59 ,,	19·7±1·4	13·9
8	35	1·99 ,,	16·8±1·8	13·5

It is seen that each zone gives an angular speed greatly in excess of that observed for sunspots, and spectrographic measures of the reversing

layer give still smaller results, that is about 5 per cent. less than the sunspot values. The relatively high value of ξ at 8° and the low value at 18° is mainly due to the eastern prominences, which have been more erratic in their movements than the western. Taking the western alone, and subtracting the average shift to red given by all the measures, the results are more uniform ; but whether we take the western or the eastern prominences separately, or together, each zone in every case gives a rotation value in excess of the corresponding values derived from spots or the reversing layer, and it seems clear from these different groupings that the accidental movements tend to cancel out even in a quite small number of measures of spectra of Class A.

The probable errors in ξ are derived from the residuals of individual measures in each group, taking the mean results given by H and K. The measuring errors are therefore relatively small, and the departures from the mean values are mainly the accidental motions of the prominences.

The rapid angular rotation is apparently not confirmed by direct observation of prominences at the limb, for in the case of those that outlast one or more rotations they may be observed to recur on opposite limbs after intervals of approximately 13 or 14 days, in agreement with spots, and this is sometimes clearly shown when a prominence appears as an absorption marking that can be identified when near the meridian in successive rotations. Such a marking was photographed at Kodaikanal in the year 1910, and observations during three successive apparitions gave a mean daily speed of 14°·4, in close agreement with spots. Yet measures of the same marking from day to day indicated a considerably larger angular speed. The marking was also intermittent in visibility, disappearing completely after some days and then reappearing in a more easterly position.* This seems to indicate that it is the point of origin of a prominence that rotates with the speed of the photosphere, while the prominence material is driven westward over the Sun, like smoke from a chimney.

An interesting case of a prominence detached from the Sun and floating entirely free at a height of 45,000 miles above the photosphere was observed on the 23rd and 24th of August 1927, on the north-west limb. The densest part of the cloud gave sharp undistorted lines (Class A), whilst in the highest portion the lines were much widened and diffused. Measures of the sharp lines gave the following results, after subtracting the general shift towards red, and adding the usual corrections :—

ϕ.	H.	V.	ξ.
+18°	100″	+3·06 km./sec.	21°

So far as could be judged on both days, it had no appreciable ascending motion, but increased in apparent altitude between the 23rd and 24th by reason of the rotation. The line-of-sight motion was somewhat greater on the 23rd. This object therefore appeared to be moving in

* *Astrophysical Journal*, **33**, 1. See also D'Azambuja, *Comptes rendus*, **176**, 950.

an orbit outside the Sun, with an angular speed considerably greater than that of the Sun, yet not nearly fast enough to maintain it in a circular orbit under gravity. The cloud could only maintain its position by radiation-pressure or other outward force. It is to be noted that in such cases, and indeed in all prominences, the atoms of hydrogen, helium, and calcium appear to be without distinction balanced against gravity: there is no segregation of these elements.

General Shift of H and K Lines in the Prominences.—The shift of the lines towards red may be shown, as a first approximation, by taking all prominences of Class A, irrespective of latitude, and subtracting the mean eastern from the mean western shift, as given in Table I. Thus we get:

	K.	H.
Half difference W−E	+·0089 A	+·0087 A
Mean . . .	+·0088 A	

This assumes that the eastern and western prominences have the same distributions in latitude, which is only approximately true. It is perhaps more instructive to group them in the four zones of Table II. Taking the differences W−E and averaging H and K, we get the following results for each zone:—

TABLE III.

Shift of Prominence Lines towards Red.

ϕ.	Mean of H and K.	No. of Spectra.
°	A.	
8	+·0073±·0021	33
18	+·0126±·0015	25
25	+·0090±·0024	26
35	+·0068±·0027	8
Weighted mean	+·0092±·0009	

The irregularities here shown are the counterpart of those in Table II., that is, they are mainly due to the eastern prominences, which give abnormally large total displacements in the 8°-zone, and abnormally small displacements in the 18°-zone. Hence the rotation shift is large and the general shift small in the first zone, and the reverse in the second. The probable errors are derived as in Table II.

This result appears to agree, within the limits of observational uncertainty, with the relativity shift, which is about +0·008 A, but it depends upon the assumed wave-lengths of the terrestrial H and K lines, which were measured at atmospheric pressure. We can only suppose the prominence lines to be emitted at zero pressure, consequently a small correction is necessary in the direction of increasing the displacement. According to the measures of W. J. Humphreys at 42, 69, and 101 atmospheres,* the mean shift of H and K is +·0017 A per atmosphere. Adopting this, we get:

Mean shift of H and K in prominences = + 0·0109 A.

* *Astrophysical Journal*, **26**, 21.

The predicted relativity shift is +0·00835 A at the Sun's surface. At 28″ above the surface it reduces to +0·0081 A. There is an outstanding difference, therefore, of 0·0028 A, which seems larger than can be accounted for by errors in the observations.

In the year 1921, a number of limb spectra of the H and K region was obtained in the third and fourth orders of the large Anderson grating at Kodaikanal, and measures of these plates gave results for the chromosphere which are almost in agreement with the above value of the shift in prominences. Thus, for H and K in the chromosphere I obtained the following :—

	K.	H.	Mean of K and H.
10 plates Sun–arc, May 1921	+0·0105	+0·0090	+0·0097 A
Add for ¾ atmosphere at Kodaikanal			+0·0013
			+0·0110 A

This value depends on the direct measurement of the shifts with reference to the calcium arc, but the measuring errors are likely to be greater than in the prominence measures, because of the width of the chromosphere lines.

For the iron lines in the reversing layer at the limb the shifts are slightly larger. The following mean shifts were obtained from the same series of plates :—

	Limb–Arc
20 Fe lines, 3885 to 3977 inclusive .	+·0102 A
Add for ¾ atmosphere	+·0014
	+·0116 A

There is very little variation in these 20 lines, only 4 giving values appreciably different from the mean.

It would seem from these results that prominences, chromosphere, and reversing layer, in this spectral region, all conspire in giving values of the shift in excess of the Einstein effect. It is evident, however, that another year's work is called for to reduce the uncertainties due to the irregular movements of the prominences. There is a possibility also of a small error, not exceeding 0·001A, in the assumed relative wave-lengths of H and K and the Fe lines to which they are referred. For this reason it would be better to measure the shift directly, between the prominence lines and the terrestrial calcium lines, and this would simplify the work. In the Fe arc the calcium lines are present, but weak, and there is an iron line so near to K that in most spectra it is blended with it. The carbon arc gives a more satisfactory calcium spectrum, and this will be used in the next series of prominence spectra.

Prominence Spectra of Class B.—Of the total number of spectra photographed, 93 are of Class B, and more than two-thirds of these are of eastern prominences. As in the case of Class A, the eastern spectra show a greater range of displacement than the western, the total range of velocity being 29 km./sec. for the eastern, and

13 km./sec. for the western spectra. The small shifts due to rotation and the Einstein effect are almost entirely swamped by these large velocities, and so no useful results can be obtained by an analysis of the measures. There are, however, some very interesting features in many of these spectra which will be discussed when a sufficient number of plates has been secured.

Ewhurst:
1927 *November* 24.

The Conditions of Emission of Forbidden Lines.

By A. S. Eddington, F.R.S.

In announcing the discovery of forbidden lines in nebulæ, Dr. Bowen pointed out that their occurrence depends on the free path being so great that the atom is unlikely to have a collision during the full duration of the metastable state. Nevertheless, it is generally felt that some further explanation is needed. All with whom I have discussed the subject have been impressed by the apparent difficulty of accounting for the great brightness of the forbidden lines in comparison with ordinary lines. A recent paper by J. Woltjer * is written from this standpoint. Although I at first shared this opinion, I have now come to the conclusion that we have been approaching the problem too elaborately, and that all that is needed is a rather obvious addendum to Bowen's original condition :

The stimulating radiation must be so weak that the atom is unlikely to absorb a quantum during the full duration of the metastable state.

Let (2) be a metastable state, the forbidden transition being $2 \rightarrow 1$. Let (3) be the state next above (2), from which there is an ordinary transition to (2). For a definite example, suppose that the full duration of (2) is 1 sec. and of (3) is 10^{-8} sec.

First, let the radiation † be moderately strong so that each atom (whether in the normal or excited state) absorbs about once in 10^{-3} sec. Then practically all the atoms arriving by any route at state (3) will forthwith emit, only 1 in 10^5 being arrested and jerked to some higher state by absorption. A fixed proportion will emit the line $3 \rightarrow 2$ and reach state (2). Of these, only 1 in 1000 will go on to state (1); the others are forestalled by absorption which diverts them to higher states. Accordingly the forbidden emission in $2 \rightarrow 1$ is very much less than ordinary emission.

Next, let the radiation be so weak that each atom absorbs about once in 10 seconds. As before, the atoms arriving at state (3) will forthwith emit, a fixed proportion of them travelling to state (2). But

* *Bull. Ast. Inst. Netherlands,* No. 138.

† The usual theory that the light of a nebula is due to stimulation by radiation from stars contained in it is here followed. Alternatives are discussed later in the paper.

now $\frac{9}{10}$ of these will go on to state (1), because the chance of absorption (once in 10 seconds) is only $\frac{1}{10}$ of the chance of emission (once in 1 second). There will be more quanta emitted in the forbidden line $2 \rightarrow 1$ than in the ordinary line $3 \rightarrow 2$, because (2) is the end-point of other transitions besides $3 \rightarrow 2$, and $\frac{9}{10}$ of *all* of them are followed by $2 \rightarrow 1$.

The strengthening of the forbidden emission is *relative* to the ordinary emission; there is no absolute strengthening. Thus if the moderately strong radiation first considered is diluted tenfold, excitation of all kinds occurs ten times less often and the ordinary emission is diminished tenfold; the forbidden emission is unaltered, because the tenfold decrease in the number of atoms reaching state (2) is compensated by a tenfold increase in the chance that any one of these will be left free to make the downward jump. The great extension of the nebula (comparable with a parsec) plays an important part in the phenomenon, because it enables a strong line to be built up from very weak emission.

Weak stimulation coupled with low density accordingly has the effect of eliminating the main cause of difference of intensity of forbidden and ordinary lines; but it would not be true to say that the emission in $2 \rightarrow 1$ attains the same intensity as if it were unforbidden. Practically no converse transitions $1 \rightarrow 2$ occur; these (if permitted) would greatly have increased the number of atoms arriving at state (2). The loss is especially heavy if the source of the stimulating radiation is a low-temperature star, rich in frequencies sufficing for the energy-difference $1 \rightarrow 2$, but poor in the extreme ultra-violet radiation required for $1 \rightarrow 3$ or higher excitations. In this connection it is to be noticed that only stars of high temperature are observed to stimulate nebular light. One of the principal factors governing the intensity of ordinary emission lines is the energy of the upper state concerned, the lines starting from higher energy levels being handicapped by the rarity of sufficient excitation. In assimilating forbidden lines with ordinary lines (under these conditions of weak stimulation and low density) we must apply the proper handicap, that is to say:

A forbidden line starting from state (2) *becomes more or less equivalent to an ordinary line starting from state* (3).

The ordinary line $3 \rightarrow 2$ is not an observable line, being too far in the ultra-violet; but it serves as a convenient link between the forbidden lines observed and the ordinary lines that are in the observable part of the spectrum. The line $3 \rightarrow 2$ would, I think, be stronger than any of the latter, so that in showing that $2 \rightarrow 1$ can become stronger than $3 \rightarrow 2$ we can conclude *a fortiori* that it is stronger than the ordinary lines of the observable spectrum.

I know no reason for anticipating that any difficulty will remain outstanding when the intensities of the lines actually observed in the nebulæ are compared according to this theory. This, however, must be left to expert spectroscopists to examine. It should ultimately be possible to make an accurate comparison of theory and observation, because the conditions in a nebula are exceptionally simple. The nebulæ are transparent to all their observed emission lines, so that the observed intensity of a line measures the total emission in the nebula—

a simplification which ought to make spectrophotometric study of the nebulæ particularly attractive and valuable. The reason for this statement is as follows.

The spectral lines in the nebulæ are of three classes : (1) ordinary principal lines, (2) ordinary subordinate lines, (3) forbidden lines. I think I am right in saying that none of the first class are in the observable part of the spectrum ; but it is absorption of these lines which provides the stimulation ultimately resulting in the nebular radiation. There will probably be moderate absorption in their wave-lengths ; but there is grave objection to accepting great optical depth of the nebula even for principal lines, because that would involve too rapid a falling off of the brightness of the nebula as we recede from the star which stimulates it. Subordinate lines are absorbed only by excited atoms, which, since the stimulating radiation is very dilute, are very rare. The forbidden lines have by definition very low emission and absorption coefficients compared with ordinary lines. Thus the absorption for classes (2) and (3) is insignificant compared with that for class (1). Any appreciable absorption of the observed emission lines would therefore involve so high an absorption for the stimulating radiation that it would be unable to make headway through the nebula.

Dr. Woltjer's theory assumes great optical depth for the observed ordinary lines, and so do most other suggestions that I have heard discussed. These theories seem to be untenable for the above reason, and do not even survive as a minor complication in the theory we are now following.

The requirement that stimulating radiation should be weak has a bearing on the question whether coronium can be a forbidden emission. The density in the corona may be low enough ; but the stimulating radiation is very intense, being practically undiluted black-body radiation. I do not think that the difficulty is evaded by assuming that the metastable state is not prematurely disturbed because the transition $2 \rightarrow 3$ requires high-frequency radiation lacking in the sun ; radiation of still higher frequency is required for the excitation $1 \rightarrow 3$, without which the forbidden line cannot be emitted. In comparing coronium with nebulium we have to remember that the former is given as a strong line by a depth of less than a million kilometres of material, whereas the latter results from a depth comparable with a parsec. According to Milne, chromospheric atoms absorb and emit about 20,000 times a second, and the coronium atoms cannot afford to be much more idle if they are to provide light of comparable intensity. This makes it unlikely that coronium can consist of forbidden lines ; at any rate, if a metastable state is concerned it must be a relatively short-lived one.

The following numerical calculation may serve as a basis for ideas as to order of magnitude. Suppose that a nebula contains an amount of oxygen equal to the sun's mass, and that each oxygen atom emits nebulium light *once a century*. The total brightness (of nebulium) will then be about 100 times the sun's brightness—equivalent to a star of absolute magnitude zero.

In the foregoing explanation I have passed over a few points of

detail which do not affect the main argument. Atoms may reach or leave a given state by capturing or expelling an electron; this is included in my argument if an ionised state is regarded as a limit of a state of high excitation, and only verbal modifications are needed. In planetary nebulæ it is doubtful whether all the stimulation comes from the central star, which, as seen from the margin of the nebulæ, would scarcely give more light than the general star-lit sky; but if the stimulation is from without instead of from within, the appropriate changes in the wording are easily made.

Collision Hypothesis.—Atoms can be stimulated to emit light either by absorbing radiation or by collision with free electrons (or possibly with other atoms). I have here followed the theory which seems to be most commonly adopted, viz. that the stimulation of the nebula atoms is due to absorption of radiation which comes (at any rate in part) from the star or stars contained in the nebula. A conceivable alternative is that the stimulation is due to collisions. These will be very rare; but, as we have just seen, the atom need only collide about once a century in order to give the observed brightness of the nebula. We have to distinguish two kinds of collision: (1) those due to the ordinary temperature movements of the nebular gas (presumed to be about 10,000°), (2) those due to streams of particles expelled by radiation-pressure or otherwise from the stars and travelling with speed much greater than the temperature speed. It is to be noticed that transitions such as $2 \rightarrow 1$ or $1 \rightarrow 2$ are not forbidden so far as collisions are concerned; in fact, it was part of Bowen's argument that, when collisions are frequent, the usual fate of an atom in state (2) is to be de-excited by collision without radiating.

The theory that the nebular light is due to ordinary temperature collisions would (if it were otherwise tenable) immediately account for the strength of the forbidden lines. The temperature is insufficient to cause emission of ordinary lines by the ions of O^{++}, O^{+}, N^{+} present in the nebula, which require high voltage. On the other hand the energy of collision is sufficient to cause transition from the normal to the low metastable states such as (2). In this case the question of the duration of state (2) in relation to the frequency of collision does not come in. If collisions are frequent a large number of atoms per second will reach state (2), but a large proportion of these will be de-excited (by collisions) without radiating; if collisions are rare, few atoms per second will reach state (2), but most of these will be allowed to radiate. Either way there will be a weak emission of forbidden lines (but practically none of ordinary lines), and the great extension of the nebula furnishes the rest of the explanation.

But I do not think that this theory of nebular light, as a purely temperature radiation, is tenable. Firstly, it leaves out of consideration what maintains the high temperature and high state of ionisation of the nebula gas—which cannot go on radiating nebulium light indefinitely without replenishment. Secondly, the occurrence of Balmer lines of hydrogen, and particularly the line 4686 of helium, shows that stimulation must be occurring much stronger than mere temperature stimulation. Whilst the high voltage stimulation required is usually

attributed to ultra-violet radiation from the stars, it could also be attributed to corpuscles emitted by them. There is, however, the objection that corpuscular emission is very uneconomical compared with radiant emission of energy, and it is almost inconceivable that the outflow of corpuscles from a star should be so great as to rival the outflow of ultra-violet radiation. A second difficulty is that the chance of an atom encountering one of the high-speed newcomers would be very rare compared with the chance of meeting an old inhabitant of low speed ; and it would be difficult to assume conditions that would give the high-speed collisions the required importance. Moreover, in the variable nebulæ the rapid changes seem to indicate a stimulus travelling with the speed of light rather than the much lower speed of, say, 50-volt electrons. For these reasons I do not favour a collision explanation of the light of a nebula ; but if, nevertheless, such an explanation is preferred, the general argument of the earlier part of this paper applies with only verbal modifications, and the observed strength of the forbidden lines does not seem to occasion any difficulty.

Excitation and Ionisation.—The question has sometimes been asked, Ought not the ordinary lines of O^{+} to be prominent, because the energy required to excite them is less than the energy required to ionise O^{+} into O^{++} ? The occurrence of the ionisation is of course manifested by the strong forbidden lines of O^{++}. We may first notice the minor point that ionisation is performed by a continuous stretch of spectrum, so that it is somewhat favoured as compared with excitation which is performed by narrow line-absorption. But setting aside this advantage, let us grant that emission of an ordinary line by O^{+} is more frequent than its ionisation. There is still another factor to take into account : once the ionisation has occurred, the O^{++} ion will go on emitting its forbidden lines at appropriate intervals until it captures an electron, and the chance of capture depends on the density of the nebula. There is no rule that each emission of a forbidden line of O^{++} must have been preceded by ionisation ; at low enough density a single ionisation may be followed by thousands of emissions.

For example, suppose that O^{+} emits an ordinary line once a month and is ionised once in 100 months. The crude argument is that any kind of O^{++} emission can have only $\frac{1}{100}$ of the strength of the ordinary line of O^{+}. This kind of argument is quite invalid. The emission from the ionised atom will be settled, *inter alia*, by its length of life. If an O^{++} ion captures an electron once in 100 months, the number of O^{+} and O^{++} ions will always be equal. We have in this problem four rates to consider: (1) rate of emission of ordinary lines by an O^{+} ion, (2) rate of emission of forbidden lines by an O^{++} ion, (3) rate of ionisation of an O^{+} ion, (4) rate of capture by an O^{++} ion. The ratio of (1) to (2) is important, and so also is the ratio of (3) to (4) ; but the ratio of (1) to (3) considered in the crude argument is irrelevant. We note that of these rates only (4) depends on the density of the nebula, and if the density is at our disposal, any desired ratio of intensity of the two sets of lines can be accounted for.

On the Integral Equation for Radiative Equilibrium.

By E. Freundlich, E. Hopf, and U. Wegner.

(*Communicated by E. A. Milne.*)

In a series of papers the Russian authors Kostitzyn,* Parchomenko,† Ambarzumian and Kosirev ‡ have discussed afresh the problem of the radiative equilibrium of a stellar atmosphere, and have attempted to bring forward a proof that the homogeneous integral equation of radiative equilibrium possesses no solution other than one identically equal to zero. These results, which are in contradiction to all solutions hitherto given, were of a nature to evoke considerable uncertainty, since, on the one hand, approximate solutions of the integral equation in question have been given by Schwarzschild, Jeans, and Milne with varying degrees of approximation to the exact solution, and, on the other hand, the testing of the theory by observation—the darkening of the solar disc towards the limb—has produced a notable confirmation of the theory. It appears further incomprehensible that a physical problem so clearly formulated as that of the transport of energy in radiative equilibrium should possess no solution. The treatment of the question by the Russian authors differs from previous treatments in that they make use of the general methods of existence theorems for the existence of solutions of homogeneous integral equations, whilst hitherto investigators have attempted to obtain approximate solutions to the integral equation or to its equivalent differential equation.

It will be shown in the following that the results of the Russian authors do not hold good and that the proof given by one of us § of the existence of the physically requisite solution is conclusive. Since a detailed discussion of the whole problem discloses some interesting relationships, we will examine here the whole question.

The integral equation of radiative equilibrium runs

$$\mathrm{B}(\tau) - \tfrac{1}{2}\int_0^\infty \mathrm{B}(t)\mathrm{L}i(e^{-|\tau-t|})dt = 0 \qquad . \qquad . \qquad (1)$$

where $\mathrm{B}(\tau)$ denotes the intensity of black-body radiation for the temperature at the optical depth τ, and $\mathrm{L}i(e^{-|\tau-t|})$, the kernel of the integral equation, denotes the logarithmic integral

$$\mathrm{L}i(e^{-|\tau-t|}) = \int_1^\infty \frac{e^{-\eta|\tau-t|}}{\eta}d\eta.$$

The integral equation (1) is a "singular" equation, since the interval of integration is infinite; owing to this, difficulties of a special kind arise in the handling of the equation. In itself the physical problem does not of necessity lead to a singular equation, since the total thickness

* *Proc. Math. Moscov*, 31; and *Ann. d. Astrophys. Instit. Moscov*, Bd. 2.
† *Ast. Nach.*, **227**, No. 5443.
‡ *M.N.*, **87**, 209–215 (1927).
§ E. Hopf, *Zeit. für Phys.* (in course of publication).

of the stellar layers is finite: nevertheless, the introduction of an infinite integration-interval avoids both any need to determine the conditions ruling at the lower boundary of the photosphere, and any need to fix what is in effect an arbitrary depth to the solar atmosphere.

So far as we are aware, it was Kostitzyn who was the first person (in a general work, *Integral Equations with Logarithmic-Integral Kernels*, and in particular in § 11 of this work) to attempt to bring forward a proof that the homogeneous integral equation of radiative equilibrium of Schwarzschild, Jeans, and Milne has no solution.

He compares equation (1) with the non-singular equation

$$B(\tau) = \int_0^H B(t)\mathrm{L}i(e^{-|\tau-t|})dt \quad . \quad . \quad . \quad (2)$$

deduced for an atmosphere of thickness H subject to the following boundary conditions:—

(1) At the external surface there is to be no radiation incident from outside;
(2) At the internal boundary—the photosphere—there is to be no emergent radiation.

It is now proved that the non-singular integral equation (2) possesses no solution other than the identically vanishing one $B(\tau)\equiv 0$. This result is correct, and (as shown by one of us) * can be extended to the case of the singular integral equation if only bounded solutions are required. But physically, for a layer of infinite thickness there is no question of bounded solutions, since the temperature in the interior increases continuously beyond limit for an infinite layer.

Kostitzyn now applies this reasoning to obtain the transition to the case of the singular homogeneous integral equation, and argues as follows: Since the integral equation (2) possesses no solutions save $B(\tau) \equiv 0$ for arbitrarily large values of H, there is no reason why in the limit $H\to\infty$ the equation should have a solution different from zero. Hence the integral equation (1) has no solution other than $B(\tau) = 0$.

This conclusion is false. For, as one of us (Hopf) has shown, the singular integral equation (1) is the limit as $H\to\infty$ of the non-homogeneous integral equation

$$B(\tau) - \tfrac{1}{2}\int_0^H B(t)\mathrm{L}i(e^{-|\tau-t|})dt = \tfrac{1}{2}K_1[H-\tau] \quad . \quad . \quad (3)$$

which possesses a solution not vanishing identically. The latter equation was discussed by Schwarzschild in his memoir, "Über Diffusion und Absorption in der Sonnenatmosphäre." † The boundary condition at the internal boundary of the photosphere as used by Kostitzyn is not physically realisable: it does not correspond to the mechanism of energy-transport implied by radiative equilibrium. For if black-body radiation of given intensity is incident on the lower boundary of the atmosphere from below, a certain proportion of this radiation is lost as back

* U. Wegner, *Zeit. für Phys.*, **45**, 808 and 829.
† *Berlin Sitz.*, 1914, equation (13).

radiation from the layer downwards : thus the flux of energy outwards from the external boundary is different from, and in fact less than, that incident on the internal boundary from below. The problem of the heating of an atmosphere of finite thickness from below leads accordingly to a non-homogeneous integral equation.

Schuster's approximate solution of equation (3) was employed by Schwarzschild to formulate the exact solution in the form

$$B_H(\tau) = \frac{0{\cdot}5 + \tau + L_H(\tau)}{H + 1}.$$

As has been shown by Hopf, in the limit $H \to \infty$ the function

$$(H + 1)B_H(\tau) = 0{\cdot}5 + \tau + L_H(\tau)$$

tends to the solution of the homogeneous singular integral equation (1), which is formally equivalent to letting $H \to \infty$ in equation (3).

The error in Kostitzyn's procedure thus consists in an erroneous passage to the limit in considering the homogeneous singular integral equation as derived from the homogeneous non-singular integral equation.

Ambarzumian and Kosirev consider directly the singular integral equation (1), and arrive at the same conclusions as Kostitzyn by the application of Fredholm's theorem in the theory of integral equations.

Clearly, however, it has escaped these authors that this theorem loses its validity for our singular integral equation. Ambarzumian and Kosirev conclude that for every solution $B(\tau)$ the integral

$$\int_0^\infty B(\tau)\psi_1(\tau)d\tau$$

must vanish. The proof of this is, however, fallacious, since the integral

$$\int_0^\infty \phi(\tau)B(\tau)d\tau$$

(equation (22), p. 214) in general does not exist : it exists only under very restricting hypotheses concerning $B(\tau)$. Both authors have quite overlooked that it is only legitimate to conclude that no solution $B(\tau)$ can exist which satisfies these restrictions. The true solution $B(\tau)$ does not in fact satisfy these restrictions. Apart from other errors, this is the principal fallacy in the considerations of Ambarzumian and Kosirev.

After this discussion it is scarcely necessary to go into the results of Parchomenko in detail, since these are based essentially on the incorrect results of Kostitzyn. The authoress attempts in particular, assuming the homogeneous equation to be without solution and starting from the non-homogeneous equation as giving the mechanism for the transfer of energy in the solar atmosphere, to replace radiative equilibrium by a state in which convection currents take an appreciable share in this transfer. She makes the assumption that the homogeneous equation has no solution, but supposes that the non-homogeneous equation—obtained from equation (1) by replacing the right-hand side by a non-vanishing function of τ—possesses a non-vanishing solution,

and evaluates the share taken by convection in the transfer of energy. Apart from the fact that there is no possibility of making any quantitative assertions about the function occurring on the right-hand side of the equation, so that Parchomenko's estimates remain quite obscure, her considerations run up against the fact that the hypothesis from which she starts has been proved to be incorrect. The work of Hopf teaches conclusively that the homogeneous integral equation possesses a solution, and that the solution found by Milne is an excellent approximation to it. In saying this much we naturally make no assertion as to the extent to which convection currents in the outermost layers of the sun and of the stars contribute to the transfer of energy. The assumption of radiative equilibrium is, of course, an idealisation of the problem. But the answer to this question cannot be tackled by the method which Parchomenko has attempted to use.

Now that a general proof of the existence of the solution of the homogeneous singular integral equation of radiative equilibrium has been brought forward, we can henceforth consider it as conclusively established that radiative equilibrium gives a possible mechanism of energy-transport in the sun and stars—indeed, almost certainly the only significant mechanism.

Einstein Institute, Potsdam :
1927 *November.*

Note on the Height of the Chromosphere as shown on the Colour Film taken at Giggleswick, 1927 *June* 29.

(*Communicated by the Astronomer Royal.*)

The British Polychromide Co., Ltd., took a colour film of the total eclipse of the sun at Giggleswick on 1927 June 29, making exposures every second and increasing the exposure gradually to about 0·5 second as totality approached. A detailed study of the separate exposures yields results of some interest.

For 27 of the exposures it may be said that the eclipse was total, and for an additional two exposures at each end two Baily's beads are shown. From 30 seconds before the beginning of totality the corona and prominences can be traced on the limb of the sun opposite to the crescent, and the same phenomena can be traced as long after totality. The corona shows as a somewhat bluish girdle round the moon during totality. Outside totality this bluish colour can also be seen at times, but generally the coronal arc appears rather reddish, a result probably due to the process employed. The chromosphere appears quite red, while the prominences vary from a deep to a whitish red.

The sequence of the photographs is as follows :—

(1) The large red prominences in position angle 325° appeared.
(2) A semicircle of corona appeared opposite the crescent, the more or less uniform layer of the chromosphere being still covered by the moon.

(3) As lower levels of the corona were uncovered by the moon, the coronal arc increased in visibility. On some exposures it appeared relatively blue, on others relatively red. The redness is due to the process employed, as it decreased with the approach of totality right up to mid-totality. There was no real red to be seen on the west limb before totality except that mentioned in (1). The material near the solar equator in position angle 265° was definitely bluish.

(4) About 2 seconds before totality the solar crescent, less than 90° long, flattened in the middle and proceeded to split into two beads which faded away together, gradually reddening, but remaining for a second or two after totality rather whiter than the rest of the chromosphere.

(5) When totality commenced there was a continuous reddish arc of chromosphere on the east limb extending for about 130° with red prominences, but still the only red prominence on the west side was that mentioned in (1).

(6) About 9 seconds after the beginning of totality the red chromosphere had nearly disappeared from the east limb, and except for the blue corona there were only three prominences shown, viz. those in position angles 325°, 75°, and about 260°. The last was associated with coronal matter and not nearly so red as the others.

(7) By the next exposure the uniform chromosphere had broken up on the east limb, and a number of faint red prominences appeared interspersed with blue corona.

(8) With the passage of the moon across the sun the faint prominences on the east limb were soon covered, while the prominences on the west increased. A third one appeared about position angle 245°, being at the beginning redder than that at 260°.

(9) About 6 seconds before the end of totality the uniform chromosphere appeared on the west limb.

(10) The chromosphere on the west limb steadily brightened. The prominence at 260° whitened but remained larger than that at 245° till about 1 second before the end of totality, when the latter rapidly brightened, and in place of the two prominences there appeared two Baily beads close together, that at 245° being much brighter.

(11) The two beads joined and formed a crescent of the sun, and phenomena similar to those before totality took place in the reverse order.

As the excess of the moon's diameter over that of the sun was 13″·1, and the duration of totality 23 seconds, the above figures indicate that the heights of the bright, more or less uniform, chromosphere on the east and west limbs were about 5″·1 and 3″·4, or 3700 km. and 2400 km. This height is considerably less than that usually ascribed to hydrogen, but from the photographs it appears to correspond to a definite layer of the sun's atmosphere. A similar result is indicated by the fact that the uniform red chromospheric layer did not show at the sun's poles.

At the end of totality it was stated that two beads appeared—the first being definitely associated with a reddish-white prominence, the second appearing later and increasing with great rapidity. The first bead, which is associated with a prominence,* may be due to an elevation of the surface layers of the sun above their normal level, the second to a depression in the moon's surface.

These photographs indicate that results of value as to the heights of various layers in the sun may be obtained from cinematograph records of eclipses. The question of taking a well-timed film with a spectrograph may well be considered for future eclipses.

We have to express our thanks to the British Polychromide Co., Ltd., for their kindness in supplying us with copies of the individual pictures obtained at the eclipse so that they might be studied in detail.

The Constants of the Star-streams from the Groningen Proper Motions.

By W. M. Smart, M.A., D.Sc.

(*Communicated by the Director, Cambridge Observatory.*)

Summary.

1. The data are the photographic proper motions of 4245 stars measured at the Kapteyn Astronomical Laboratory, Groningen.

2. The frequency curves of the proper motions have been analysed into two drifts, and it is found that the stars are equally divided between the two drifts.

3. The positions of the drift apices are as follows :—

Drift I. : R.A. 89° ; Dec. − 3°.
Drift II. : R.A. 279° ; Dec. − 56°.

4. The co-ordinates of the vertex of star-streaming are—

R.A. 91° ; Dec. + 15°,

which is in Galactic Latitude − 1°.

5. The co-ordinates of the solar apex are—

R.A. 273° ; + 31°.

§ 1. Two years ago Dr. G. H. ten Bruggencate † took in hand the analysis of the photographic proper motions in twenty-seven regions. Of these regions eighteen are evenly distributed in right ascension along a narrow zone in declination − 45° ; the remainder are miscellaneous regions in the northern hemisphere. The photographic plates had been measured at the Kapteyn Astronomical Laboratory at

* Objective prism spectrograms, obtained by the Hamburg expedition to Jokkmokk (Lapland), show that this prominence was associated with strong continuous spectrum.

† *B.A.N.*, No. 85 (1925 Sept. 30).

Groningen, and the results published in Nos. 28, 30, 33, and 39 of the *Groningen Publications*. Bruggencate's paper gives the details of the individual regions with the probable errors of the proper motions, the number of stars, and the mean photographic magnitude. For twenty-four of the regions the average probable error of the proper motions in either co-ordinate is 0″·6 per century; in the remainder it is smaller (the average interval on which the proper motions are based is about seventeen years). All stars brighter than the ninth photographic magnitude were omitted from the analysis, and the mean photographic magnitude of the stars with which the paper is concerned varies between 11·0 and 14·0 in the various regions. Bruggencate gives the frequency curves for each region—these have been obtained by counting the numbers of the stars moving in 10° sectors of position angle and smoothing the counts by taking the mean of three adjacent sectors. With scarcely an exception the curves show up very well the effect of star-streaming. Bruggencate analysed his curves by Dyson's method * and obtained the apices of the two drifts in excellent accordance with the results hitherto obtained from similar investigations based on meridian proper motions. Dyson's method of analysis, however, does not give a complete account of the phenomena—it is unable to yield values for the drift velocities, the preferential direction of star-streaming, and the solar motion relative to the general body of stars with which the particular investigation deals. The Groningen proper motions are well over 4000 in number, and this valuable material is evidently capable of furnishing more complete information than Bruggencate obtained. A year ago I communicated to the Society the results † of an analysis of the Cambridge photographic proper motions—3029 in number—and in the present paper I submit the Groningen proper motions to a similar analysis.

§ 2. Wherever it was possible contiguous regions were grouped together so as to provide a sufficiently large number of proper motions to be analysed. Two Groningen regions were omitted—Nos. II. and III. —the latter because of a strong north polar drift which may be a real local phenomenon or the result of some systematic error. In Table I. are assembled the relevant data for the twelve groups, A . . . M, formed from the twenty-five Groningen regions. The third and fourth columns give the mean positions of the centres of the various groups weighted according to the number of the stars in the component regions. The last column gives the weight accorded to each group in the least-squares solution—this weight depends mainly on the number of stars per group, and to a lesser extent on the probable errors of the proper motions and on the comparative success obtained in representing the observed frequency curve with a theoretical two-drift solution.

§ 3. Bruggencate does not give in his paper the counts in each region of the proper motions in the different position angles, but only the frequency curves based on these counts. However, it is a simple matter to deduce from his diagrams the numbers of proper motions in the various directions—it is necessary to do this for each region, for the

* *Proc. Roy. Soc. Edin.*, **28**, 231.
† *M.N.*, **87**, 122.

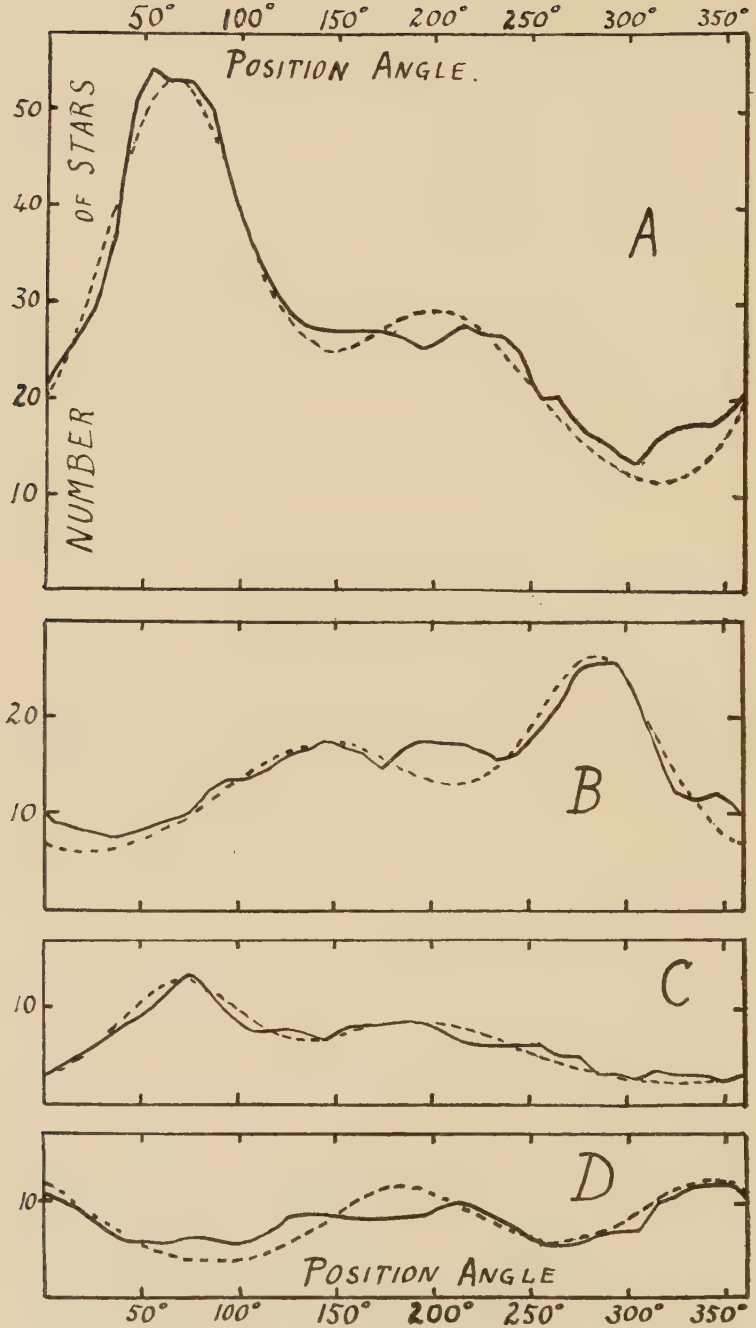

FIG. 1.—(Groups A, B, C, D.) Observed distribution of proper motions (full line); theoretical distribution (dotted line).

TABLE I.

Group.	Groningen Regions.	R.A. h m	Dec. °	Number of Stars.	Weight.
A	I., VI.	19 57	+24·4	903	3
B	IV., V.	13 11	+37·6	539	2
C	VII., VIII.	2 19	−46·6	235	1
D	IX., X., XI.	6 0	−44·6	288	1
E	XII., XIII.	8 28	−43·2	424	2
F	XIV., XV., XVI.	10 58	−45·0	342	2
G	XVII., XVIII., XIX.	13 41	−44·7	284	1
H	XX., XXI., XXII.	16 55	−44·5	374	2
J	XXIII.	19 30	−43·0	115	½
K	XXIV.	22 0	−44·0	138	½
L	XXV.	17 37	+68·0	51	½
M	XXVI., XXVII.	14 5	+34·2	684	3

curves have not all been drawn on the same scale. Table II. gives the results of this analysis, namely, the number of stars moving between position angles 0° and 10°, 10° and 20°, etc., for each of the combined groups A, B, . . . M (fractional values occur as the numbers have been smoothed by taking as the effective number moving in a sector $\theta - 5°$ to $\theta + 5°$, the mean of the observed numbers in the three sectors $\theta - 15°$ to $\theta - 5°$, $\theta - 5°$ to $\theta + 5°$, and $\theta + 5°$ to $\theta + 15°$). In the diagrams which follow, the full-line curves represent the distribution of proper motions in position angle according to the statistics of Table II.—the curve for Group A is, however, an exception, for it has been formed by giving double weight to Groningen region I., in combination with VI., in consideration of the smaller probable errors of the proper motions in the former region.

TABLE II.

Distribution of Proper Motions in Position Angle.

Position Angle. °	A.	B.	C.	D.	E.	F.	G.	H.	J.	K.	L.	M.
5	22·0	9·0	3·7	10·3	10·7	4·7	3·0	3·7	1·0	0·7	2·7	9·7
15	24·0	8·3	4·7	9·7	10·7	5·3	2·7	4·7	1·0	1·0	2·0	8·3
25	26·7	8·0	5·7	8·0	10·3	6·0	2·3	5·7	1·0	1·0	1·7	7·7
35	33·3	7·7	7·3	6·7	8·7	6·3	1·7	6·7	1·3	1·3	1·3	8·3
45	41·3	8·0	8·7	5·7	8·3	6·7	1·7	6·7	1·3	2·3	1·3	7·7
55	43·0	8·7	9·7	5·7	6·7	5·7	2·3	6·7	1·3	2·7	1·0	8·3
65	42·7	9·3	11·7	5·7	6·0	5·0	2·7	6·7	1·7	4·7	0·7	8·3
75	42·7	10·0	13·3	6·3	6·7	5·3	3·3	6·0	1·7	5·7	0·3	8·3
85	40·0	12·3	11·7	6·0	8·3	6·0	4·0	6·3	2·0	6·3	0·3	8·3
95	33·7	13·3	9·3	5·7	9·3	6·3	4·7	6·7	3·0	7·3	0·3	9·0
105	28·7	13·7	7·7	5·7	9·3	7·3	5·7	7·3	4·0	8·3	0·3	11·3

TABLE II.—*continued.*

Position Angle.	A.	B.	C.	D.	E.	F.	G.	H.	J.	K.	L.	M.
°												
115	27·0	14·3	7·3	6·7	9·7	8·0	6·7	7·7	4·7	8·7	0·3	19·0
125	25·3	16·0	7·7	8·3	10·7	8·3	8·3	9·3	6·3	8·3	0·7	21·3
135	24·7	16·7	7·3	8·7	12·7	8·7	8·7	12·0	8·0	7·7	1·7	22·7
145	24·3	17·3	6·7	8·7	13·3	9·0	7·7	15·3	8·7	6·7	2·3	23·3
155	24·7	17·0	7·7	8·3	13·7	9·7	6·3	15·7	7·3	5·0	2·3	22·7
165	24·3	16·3	8·0	8·0	14·3	9·3	4·3	16·0	7·0	3·7	2·3	22·0
175	24·3	14·3	8·3	8·0	15·0	5·3	4·3	17·7	7·0	3·3	2·3	21·0
185	23·7	16·7	8·3	8·3	14·7	6·0	4·7	20·7	7·0	3·0	2·3	19·7
195	22·7	17·3	8·3	8·3	13·3	7·3	6·0	20·0	5·3	3·0	2·0	17·3
205	24·0	17·3	7·3	9·3	11·0	7·7	9·3	20·7	3·3	4·0	2·0	16·0
215	25·7	17·3	6·3	9·7	10·3	7·3	12·7	24·7	3·0	4·3	1·7	16·0
225	26·0	16·7	6·0	9·0	10·7	8·0	16·0	26·0	3·0	4·3	1·3	16·7
235	25·7	15·7	6·0	8·0	11·3	10·0	19·0	23·0	2·7	4·3	1·0	18·7
245	24·3	16·3	6·0	6·7	10·7	12·7	18·7	16·3	2·7	4·3	0·3	22·7
255	19·7	18·7	6·3	5·3	10·7	18·7	19·3	13·3	1·3	4·3	0·3	28·0
265	19·3	21·0	5·0	5·3	11·3	23·3	19·0	11·7	1·3	3·7	0·3	32·3
275	15·7	24·7	5·0	5·7	14·7	22·7	17·3	11·0	1·3	3·3	0·3	35·3
285	15·0	25·3	3·0	6·7	15·3	20·7	14·7	7·0	1·3	3·3	0·3	35·3
295	13·3	25·7	3·0	7·0	16·0	18·0	10·7	4·0	1·3	2·0	0·3	34·3
305	12·3	22·3	2·7	7·0	17·0	15·3	8·3	3·0	1·7	2·0	1·3	31·3
315	15·3	17·0	3·3	10·0	18·3	12·0	6·7	3·0	1·7	2·3	1·7	28·7
325	16·3	12·3	3·0	10·7	19·0	8·7	6·0	3·0	1·7	2·3	2·3	27·0
335	16·7	11·3	3·0	11·7	13·7	6·0	5·7	2·7	1·7	1·0	3·0	22·0
345	16·7	12·0	2·7	12·0	10·7	5·7	5·0	3·3	1·3	1·0	3·3	15·5
355	18·3	11·0	2·7	11·7	10·3	4·7	3·7	3·7	1·3	1·0	3·3	12·0

§ 4. As in the analysis of the Cambridge photographic proper motions, Professor Eddington's method * of resolving the observed frequency curve into two component drift curves has been followed. Briefly, if the numbers of the stars in the two drifts are N_1 and N_2, if the drifts are pointing in position angles θ_1 and θ_2, and if the projected drift-velocities are $h\mathrm{V}_1$ and $h\mathrm{V}_2$ (in terms of a theoretical unit $1/h$), then the total number (r) of stars with proper motions between position angles $\theta - 5°$ and $\theta + 5°$ is given † by

$$r = 0{\cdot}0492\mathrm{N}_1\phi(h\mathrm{V}_1, \theta - \theta_1) + 0{\cdot}0492\mathrm{N}_2\phi(h\mathrm{V}_2, \theta - \theta_2),$$

in which the values of the function ϕ are tabulated in the paper just quoted. For successive values of θ, the observed frequency curve provides the corresponding values of r, and the analysis consists in deriving (by trial and error) the values of the six drift constants

* *M.N.*, **67**, 34.

† *Ibid.*, **87**, 127.

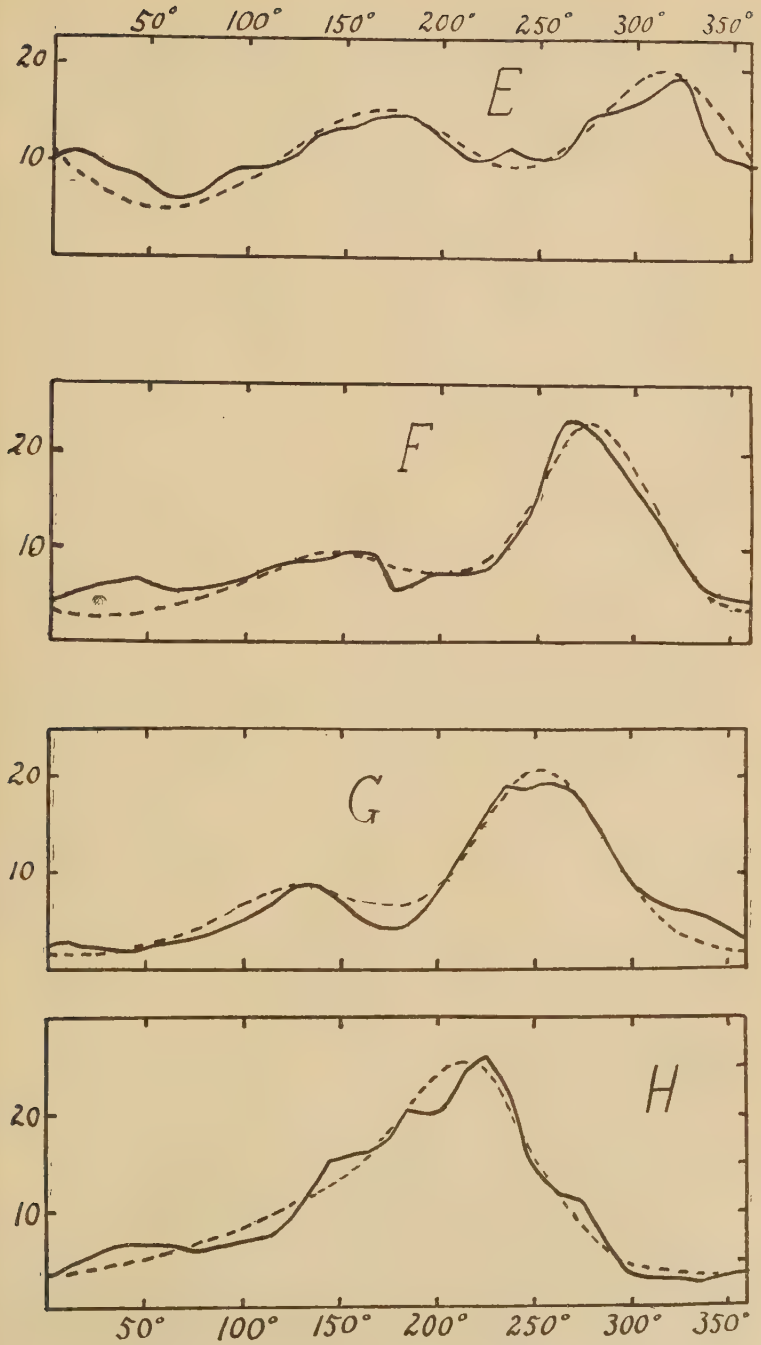

FIG. 2.—(Groups E, F, G, H.) Observed distribution of proper motions (full line); theoretical distribution (dotted line).

N_1, N_2, . . . hV_2, which give the best fit with the observed curve. Table III. summarises the results of the analysis. (The curve for

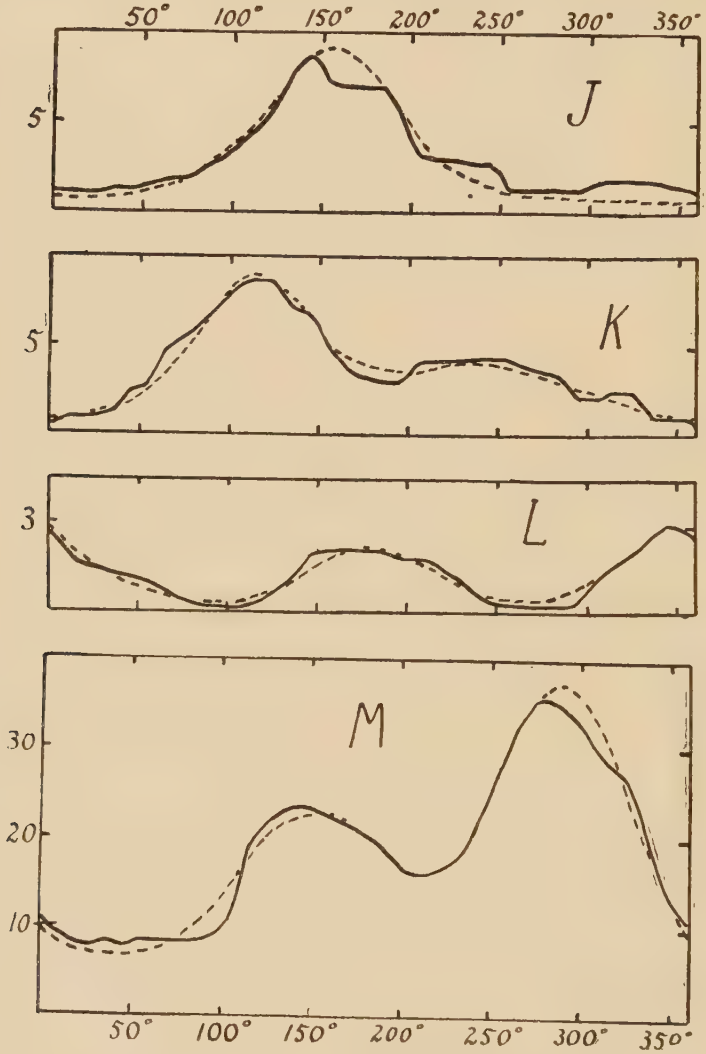

FIG. 3.—(Groups J, K, L, M.) Observed distribution of proper motions (full line); theoretical distribution (dotted line).

Group J has been taken to be a single drift curve—the centre of the region is close to the apex of Drift II.) In the following diagrams (position angle θ as abscissa and the number r of stars as ordinate) the full-line curves are the observed frequency curves and the broken

curves are the theoretical curves derived by means of the equation above for r from the data of Table III.

TABLE III.

The Values of the Drift Constants for the Twelve Regions.

Group.	N_1.	N_2.	θ_1.	θ_2.	hV_1.	hV_2.
A	484	419	65°	205°	0·90	0·50
B	229	310	285	145	1·10	0·45
C	95	140	70	195	1·20	0·50
D	150	138	345	185°	0·75	0·75
E	200	224	315	165	0·90	0·60
F	170	155	280	150	1·30	0·50
G	180	104	255	130	1·15	0·75
H	197	177	215	135	1·05	0·35
J	105	..	155	..	0·85	..
K	66	72	115	235	1·30	0·45
L	26	25	350	175	1·30	0·80
M	360	324	290	150	1·00	0·60

§ 5. Let X, Y, Z denote the components (referred to equatorial rectangular axes) of the linear velocity in space of Drift I.: (α, δ) the co-ordinates of the centre of a group (these are given in Table I.). Then each group furnishes the pair of equations in X, Y, and Z:—

$$\begin{aligned} -\mathrm{X}\sin\alpha + \mathrm{Y}\cos\alpha &= +hV_1\sin\theta_1 \\ \mathrm{X}\cos\alpha\sin\delta + \mathrm{Y}\sin\alpha\sin\delta - \mathrm{Z}\cos\delta &= -hV_1\cos\theta_1 \end{aligned}$$

the values of hV_1 and θ_1 being taken from Table III. There are similar equations for Drift II. Solutions by least squares yield the following results:—

	Drift I.	Drift II.
X	+0·019±0·033	+0·053±0·028
Y	+1·247±0·033	−0·332±0·028
Z	−0·072±0·036	−0·506±0·031

From these values of X, Y, Z the co-ordinates of the drift apices and the space velocities (relative to the sun) of the drifts—in terms of the unit $1/h$—are easily deduced; they are:

	Drift I.	Drift II.
R.A. of Apex	89°·1±1°·5	279°·0±15°·7
Dec. „	−3°·3±1°·7	−56°·5±2°·7
Space Velocity (hV)	1·249±0·033	0·607±0·030

The following table gives the values of the constants from certain investigations, the first line from the meridian proper motions of Boss's Catalogue, the second line from the Cambridge photographic proper motions, the third line contains Bruggencate's results (limited

to the co-ordinates of the apices), and the fourth line the results of the present analysis :—

TABLE IV.

	Drift I.			Drift II.		
	Apex		Velocity (kV).	Apex		Velocity (kV).
	α.	δ.		α.	δ.	
	°	°		°	°	
Eddington (Boss proper motions)	91	−15	1·52	288	−64	0·86
Smart (Cambridge proper motions)	88	−12	1·50	289	−73	0·97
Bruggencate (Groningen proper motions)	88	−10	..	300	−50	..
Smart (Groningen proper motions)	89	− 3	1·25	279	−56	0·61

Consider the last two lines of Table IV. which give the results of analysis of the same material. Taking Drift II. first, the two separate determinations of the apex are reasonably accordant when the nature of the computed probable errors are taken into account. As regards Drift I., the most notable difference concerns the declination of the apex, the value derived in this paper being the lowest ever found in similar investigations. The two drifts are generally well separated in the Groningen regions and it is not easy to see how the divergence between Bruggencate's value for the declination of Drift I. apex and the value found in this paper can arise.

The maxima of the two drifts in the Groningen frequency curves are somewhat flatter than in the Cambridge curves—this is probably due in part to the larger probable errors of the proper motions in the former—and this difference is shown up in the smaller values of the space-velocities of the two drifts.

§ 6. Using the position of Drift I. apex, as derived in this paper, we can calculate the position angle θ_1 of the drift for each of the twelve groups A . . . M, and then compare this value with that actually derived from the analysis of the particular frequency curve. In Table V. below the second column gives this difference for θ in the sense "observed−calculated." The fifth column gives analogous information for Drift II. Again, if kV denotes the space velocity of Drift I., and λ_1 the angular distance between the centre of a region and the position of the Drift I. apex, then the projected drift velocity $k\mathrm{V}_1$ for any region is given by

$$k\mathrm{V}_1 = k\mathrm{V} \sin \lambda_1.$$

The differences between the values of $k\mathrm{V}_1$ calculated by this formula and the values derived from the individual frequency curves are given in the fourth column (the seventh column contains the corresponding differences with respect to Drift II.).

§ 7. We now deduce the co-ordinates of the vertex of star-streaming. Let ξ, η, ζ denote the components of the velocity of Drift I. relative to Drift II. Then from the values of X, Y, Z in § 5 we obtain

$$\begin{aligned} \xi &\equiv \mathrm{X}_1 - \mathrm{X}_2 = -0{\cdot}034 \quad \pm 0{\cdot}044 \\ \eta &\equiv \mathrm{Y}_1 - \mathrm{Y}_2 = +1{\cdot}578 \quad \pm 0{\cdot}044 \\ \zeta &\equiv \mathrm{Z}_1 - \mathrm{Z}_2 = +0{\cdot}434 \quad \pm 0{\cdot}048 \end{aligned}$$

The position of the vertex is then found ; it is :

R.A. of vertex	**91°·2**	± 1°·6
Dec. ,, ,,	**+ 15°·4**	± 1°·6

Also the velocity hW of one stream relatively to the other is 1·637 in the usual unit.

TABLE V.

Group.	Drift I. θ_1 (Obs.–Cal.).	Drift I. λ_1.	Drift I. hV$_1$ (Obs.–Cal.).	Drift II. θ_2 (Obs.–Cal.).	Drift II. λ_2.	Drift II. hV$_2$ (Obs.–Cal.).
	°	°		°	°	
A	+ 6	144	+0·16	+14	83	−0·11
B	+ 6	107	−0·10	− 7	116	−0·10
C	+ 5	64	+0·08	−19	64	−0·05
D	−13	42	−0·18	0	79	+0·15
E	+ 6	53	−0·10	0	78	0·00
F	+ 2	77	+0·08	+ 4	65	−0·05
G	+ 6	106	−0·05	− 2	46	+0·31
H	+15	130	+0·09	− 1	20	+0·14
J	+ 6	130	−0·11	..	..	..
K	− 7	114	+0·15	+ 4	34	−0·06
L	− 5	115	+0·17	+ 5	125	+0·30
M	+ 3	118	−0·10	+ 2	107	+0·02

Table VI. gives the results of several other investigations with the above results in the last line for comparison.

TABLE VI.

Vertex.

	Vertex. a.	Vertex. δ.	Velocity hW.
	°	°	
Eddington (Boss stars)	94	+12	1·87
Dyson (large proper motions)	88	+21	..
Smart (Cambridge proper motions)	92	+20	1·84
Smart (Groningen ,, ,,)	91	+15	1·64

§ 8. The analysis of the frequency curves shows that the stars are nearly equally divided in number between the two streams ; the actual numbers are 2157 for Stream I. and 2088 for Stream II. We can now derive the apex of the solar motion referred to this group of 4245 stars. The results are—

R.A. of solar apex	**272°·6**
Dec. ,, ,,	**+31°·1**

The declination is notably lower than the value (+ 43°·6) found in the case of the Cambridge proper motions. If we take the positions of the

drift apices as determined by Bruggencate and assume that the numbers in the two drifts are as found above, the declination of the solar apex comes out at practically the same value, namely $+31°$. The two methods of analysing the Groningen proper motions thus lead to almost identical values for the right ascension and declination of the solar motion.

The speed of the solar motion is found from the analysis to be 0·553 in terms of the unit $1/h$. The value found by Professor Eddington is 0·908 for the Boss stars, and that found from the Cambridge proper motions is 0·881. The Groningen proper motions thus give a low value for this constant, a consequence of the smaller values of the space-velocities of the drifts.

Observatory, Cambridge:
1927 December 5.

The Intensity of an Absorption Line in a Spectrum produced by a Grating. By J. A. Carroll, M.A., Ph.D., Assistant Director of the Cambridge Solar Physics Observatory.

(*Communicated by the Director.*)

1. Owing to the finite resolving power of a diffraction grating there falls at any point in a spectrum produced by it not only the light of the wave-length of the principal maximum of diffraction at that point (the ordinary wave-length at that place), but also light of all other wave-lengths present. Thus the intensity in the centre of an absorption line is greater than it would be in a pure spectrum, as yielded by an instrument of infinite resolving power, so that the line is filled in and also broadened.

It is possible in any given case to work out the exact contour of the line yielded by a grating, or to deduce from the observed form the true form, but no simple general relations can be found. Interest, however, often centres only upon the central intensity of the line, and the present paper shows that in practical cases the filling in is by no means negligible, amounting commonly to 10 per cent. or so, and gives a simple relation enabling the necessary corrections to be applied to deduce the true depth of the line from the observed depth with all necessary accuracy.

2. The intensity of the light of wave-length λ diffracted at an angle w from a grating of m lines, each of width ω, and of constant d, is given by the well-known relation

$$I = I_0 \left\{ \frac{\sin \frac{\mu w}{2}}{\frac{\mu w}{2}} \right\}^2 \left\{ \frac{\sin \frac{\mu m d}{2}}{\sin \frac{\mu d}{2}} \right\}^2 \quad . \quad . \quad . \quad (1)$$

$$\mu = \frac{2\pi}{\lambda} \sin \phi \quad . \quad . \quad . \quad . \quad (2)$$

Thus if the true form of the incident spectrum be

$$I = I(\lambda)\{1 - af(\lambda)\} \quad . \quad . \quad . \quad . \quad (3)$$

where $f(\lambda)$ represents the form of the absorption line and a its depth ($f(\lambda_m) = 1$ where λ_m = wave-length of centre of line), and $I(\lambda)$ the intensity of the continuous spectrum, then the grating yields at any point ϕ an intensity

$$\int_{\lambda_1}^{\lambda_2} I(\lambda)\{1 - af(\lambda)\}\left\{\frac{\sin\frac{\mu w}{2}}{\frac{\mu w}{2}}\right\}^2\left\{\frac{\sin\frac{\mu md}{2}}{\sin\frac{\mu d}{2}}\right\}^2 d\lambda \, . \quad . \quad (4)$$

Since the intensity in the subsidiary maxima rapidly decreases, it will be sufficient to suppose $I(\lambda)$ constant, and also we may neglect the variation of $\sin\frac{\mu w}{2}\Big/\frac{\mu w}{2}$; further $f(\lambda)$ also decreases very rapidly away from λ_m so that the range of integration, providing only one order of spectrum is present, is practically over a number of subsidiary maxima small compared with m. It also appears that $\int\left(\frac{\sin mx}{\sin x}\right)^2 dx$, when m has the values commonly occurring in practice, is very closely equal over the same range of integration to its value over a range of 2π in x. The precise form of $f(\lambda)$ also is not of great importance for determining the central intensity, for so long as it decreases rapidly on both sides of λ_m the contribution to the integral is small except in the immediate neighbourhood of λ_m, so that we shall consider a line of the form $e^{-\alpha^2x^2}$, where x is wave-length or frequency measured from λ_m or ν_m (frequency) as origin. The intensity at the centre of the line

$$I = I_0(1 - ae^{-\alpha^2(\nu-\nu_m)^2})$$

is then

$$I = I_0\int(1 - ae^{-\alpha^2(\nu-\nu_m)^2})\frac{\sin^2\frac{\mu md}{2}}{\sin^2\frac{\mu d}{2}}d\nu \quad . \quad . \quad (5)$$

where a is the "true depth" of the line $\left(\frac{OT}{OA}\right.$ in fig. 1$\left.\right)$. Thus the observed contrast $\left(\frac{OG}{OA}\right.$ in fig. 1$\left.\right)$ is given by

$$\frac{OG}{OA} = \frac{a\int e^{-\alpha^2(\nu-\nu_m)^2}\frac{\sin^2\frac{\mu md}{2}}{\sin^2\frac{\mu d}{2}}d\nu}{\int\frac{\sin^2\frac{\mu md}{2}}{\sin^2\frac{\mu d}{2}}d\nu}.$$

Thus if r is the ratio of the apparent depth $\left(\frac{OG}{OA}\right)$ to the true depth $\left(\frac{OT}{OA}\right)$

$$r = \frac{OG}{OT} = \frac{1}{a} \cdot \frac{OG}{OA}.$$

The centre of the line is a principal maximum for ν_m in the spectrum of order h say, so we have

$$\frac{\nu_m}{c} d \sin \phi = h$$

$$2\pi \frac{\nu}{c} \sin \phi = \mu \frac{\mu d}{2} = \pi \frac{\nu}{\nu_m} . h.$$

Put

$$\nu = \nu_m \left(1 + \frac{\xi}{h\pi}\right)$$

$$r = \frac{\int e^{-\alpha^2 \nu_m^2 \frac{\xi^2}{h^2\pi^2}} \frac{\sin^2 m\xi}{\sin^2 \xi} d\xi}{\int \frac{\sin^2 m\xi}{\sin^2 \xi} d\xi} \quad . \quad . \quad . \quad (6)$$

In practice the range of continuous spectrum present is several hundred A.U. of wave-length, and covers many subsidiary maxima, so that the range of ξ for the denominator of (6) is $0 - 2\pi$, and for the numerator $-a$ to $+a$, where a is small compared to m, so that

$$r = \frac{1}{\pi m} \int_{-a}^{a} e^{-\beta^2 \xi^2} \frac{\sin^2 m\xi}{\sin^2 \xi} d\xi \quad . \quad . \quad . \quad (7)$$

The numerical computation of r has been carried out for a grating of 100,000 lines, for any other where $m = p \,.\, 10^5$ we have, since p lies in practice between 10^{-2} and 10^2, and thus $\sin \xi = \xi$ very nearly over the range of integration in (7),

$$r = \frac{1}{10^5 \pi} \int_{-b}^{+b} e^{-\gamma^2 \eta^2} \frac{\sin^2 10^5 \eta}{\sin^2 \eta} d\eta \quad . \quad . \quad . \quad (8)$$

where

$$\beta = \alpha \nu_m / h\pi$$
$$\gamma = \alpha \nu_m / ph\pi.$$

Let R be the "resolving half width $\Delta\nu$" of the grating at the point considered in the spectrum

$$\lambda_m / \Delta\lambda = \nu_m / \Delta\nu = hp \,.\, 10^5.$$

Let W be the "half width of the line," *i.e.* the value of $\nu - \nu_m$, for which the true intensity curve is reduced to the value $1/e$ of its central value. Then

$$W = \frac{1}{\alpha}, \qquad \frac{R}{\nu_m} = \frac{1}{hp \,.\, 10^5}$$

$$\gamma = \frac{10^5}{\pi} \cdot \frac{R}{W} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Thus we can make the computation of r for different values of γ for a particular grating serve for all lines for any grating of from say 10^3 to 10^7 lines, since the results are the same whenever R/W is the same. (R and W may be measured, of course, either in λ or ν equally well.)

The results are tabulated in Table I. below.

TABLE I.

W/R.	r.	W/R.	r.
0·00	0·000	0·70	0·728
0·05	0·100	0·80	0·762
0·10	0·177	0·90	0·788
0·15	0·254	1·00	0·806
0·20	0·327	1·50	0·840
0·30	0·454	2·00	0·865
0·40	0·557	5·00	0·94
0·50	0·634	10·00	0·97
0·60	0·686	∞	1·000

For the sake of illustration, the case of a line at $\lambda_m = 5000$ A.U. in the first order of a grating of 10^5 lines has been worked out over the

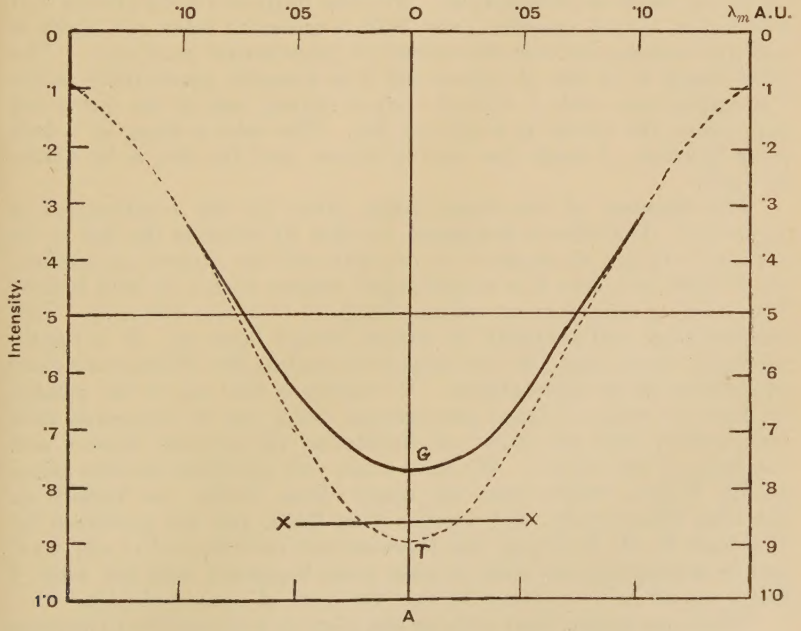

FIG. 1.

width of the line and the result shown in fig. 1. For a line of perfect blackness at the centre the grating shows an intensity of ·136 of the

neighbouring continuous spectrum; a line having a true central intensity of $\frac{1}{10}$ of the continuous background is shown as one with a central intensity of ·220.

The dotted curve represents the true form of the absorption line, the full curve the line as rendered by the grating. XX represents the residual intensity for a perfectly black line, *i.e.* when OT = OA. The curves are drawn to scale for a line half-width ·10 A.U. ($e^{-40^2(\lambda-\lambda_m)^2}$) of wave-length 5000 A.U. and a grating of 10^5 lines in the first order, and a true intensity of $\frac{1}{10}$ of the continuous spectrum.

Note on Photography and the Physical Condition of the Moon.

By H. G. Tomkins.

Some years ago I brought before the Society a suggested explanation of the origin of the Lunar Bright Rays. In the study of these it became evident, owing to the variation of illumination, and their different nature, that drawings would be of very little assistance, and I decided to try my hand at photography. For this purpose I have ground and polished a 24-inch speculum, and built a reflecting telescope which is now just coming into use after about 10 years actual work on it. The focal length is 12 feet $7\frac{1}{4}$ inches, and it is mounted equatorially in the Cassegrain form with a $6\frac{1}{2}$-inch convex mirror, and at the lower end just above the mirror is a $5\frac{1}{2}$-inch flat. The tube is hung in a fork after Ritchey. I made the convex mirror, and the flat is by Cooke of York.

The diameter of the lunar image given by the combination is $5\frac{1}{4}$ inches. The tube is hexagonal, so that by rotating the flat in its setting I can get an eyepiece on one side and the camera on another. On a third side there is a second larger camera with a $2\frac{1}{4}$-inch Barlow lens, which gives at present a lunar image of 11 inches diameter. This magnification will probably be pushed further later on. It is hardly necessary to say that I do not propose to emulate Mt. Wilson with their magnificent lunar photographs. My feeling is that up to the present we have no series of lunar photographs which can be compared with one another with the object of elucidating the physical history and condition of the surface. We have a few very excellent pictures taken at Mt. Wilson, others—also not many—from Yerkes, an Atlas from the Lick Observatory, and another from Paris, and one published by Professor W. H. Pickering, but these are not co-ordinated in any way, and in attempting last year to make some beginning with the work, I found myself held up almost at once for want of connected data.

When one realises that even on one plate it is impossible to develop for more than a limited zone of longitude in order to get really useful results, and when we find that in the Atlases there are generally only about two lightings on a zone, namely, the rising and setting sun—the Lick Atlas has only 19 plates—it is evident that little can be done.

My idea is to take zones, or even individual formations in a number of various illuminations, photographing and studying them together, so as to secure a more individual examination of them. This I feel should place in our hands much more information, of a character that cannot be questioned on the point of accuracy, than we now possess; and had such photographs been available in the past, the problem of the crater Linné, as an instance, would have been settled long ago, and there are many other problems still waiting to be solved.

1927 *December* 6.